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CHIRAL SCALAR FIELDS, ELECTROWEAK $SU(2)_L \times U(1)$ AND THE QUANTIZATION OF THE ELECTRIC CHARGE.

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Abstract: I study the scalar representations of the electroweak group of the Standard Model, with a special emphasis on their chiral properties and on their behaviour by the discrete symmetries P and CP . A gauge theory for $J = 0$ mesons naturally springs out, together with a new approach to the question of the electroweak violation of CP . When acting in the 4-dimensional space of states spanned by the special representations under scrutiny in this paper, the electric charge appears as one of the three generators of the diagonal $SU(2)$ and its eigenvalues are consequently quantized.

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1 Introduction.

While scalar fields play a crucial and ambiguous role in the spontaneously broken gauge theory of electroweak processes [1], pseudoscalar and scalar mesons are traditionally attached to the chiral group of strong interactions [2]. However, the dominance of their electroweak interactions makes mandatory their description within the framework of a gauge theory too. This is achieved in this paper, which also re-unites within the same framework the fields at the origin of the breaking of the symmetry and the observed particles (mesons).

The group $SU(2)_L \times U(1)$ of electroweak interactions has, as will be shown below, the fundamental property and advantage that it is a subgroup of the chiral group $U(N)_L \times U(N)_R$ for N even. Its generators can thus be taken as $N \times N$ matrices. Similarly, the $2N^2$ Lorentz scalar and pseudoscalar mesons fields can naturally be represented as $N \times N$ matrices. I reduce the most general $J = 0$ representations of the electroweak group to $N^2/2$ quadruplet representations.

In the space of states spanned by these representations, the electric charge becomes one of the three generators of the diagonal $SU(2)$; its eigenvalues are consequently quantized. The existence of a unique quadratic invariant for those representations also allows to write a $SU(2)_L \times U(1)$ gauge invariant Lagrangian for $J = 0$ mesons.

From their transformation properties by parity and charge conjugation we can also deduce in a straightforward way non trivial results concerning CP transformations of observed electroweak states.

2 The chiral group $U(N)_L \times U(N)_R$.

A generator \mathbb{A} of $U(N)_L \times U(N)_R$ is a set of two $N \times N$ matrices $(\mathbb{A}_L, \mathbb{A}_R)$. A generator of a diagonal subgroup satisfies $\mathbb{A}_L = \mathbb{A}_R$.

Both left and right parts of the chiral group violate parity; hence it is natural to classify the $J = 0$ fields according to their behaviour by the parity changing operator \mathcal{P} , which transforms a scalar into a pseudoscalar and vice-versa; we shall accordingly consider the action of the chiral group on \mathcal{P} -even or \mathcal{P} -odd states.

We define it by the actions of its left and right commuting subgroups. At the level of the algebra:

$$\begin{aligned} \mathbb{A}_L^i \cdot \mathbb{M}_{\mathcal{P}\text{even}} &\stackrel{\text{def}}{=} -\mathbb{A}_L^i \mathbb{M}_{\mathcal{P}\text{even}} = \frac{1}{2} ([\mathbb{M}_{\mathcal{P}\text{even}}, \mathbb{A}_L^i] - \{\mathbb{M}_{\mathcal{P}\text{even}}, \mathbb{A}_L^i\}), \\ \mathbb{A}_L^i \cdot \mathbb{M}_{\mathcal{P}\text{odd}} &\stackrel{\text{def}}{=} +\mathbb{M}_{\mathcal{P}\text{odd}} \mathbb{A}_L^i = \frac{1}{2} ([\mathbb{M}_{\mathcal{P}\text{odd}}, \mathbb{A}_L^i] + \{\mathbb{M}_{\mathcal{P}\text{odd}}, \mathbb{A}_L^i\}), \\ \mathbb{A}_R^i \cdot \mathbb{M}_{\mathcal{P}\text{even}} &\stackrel{\text{def}}{=} +\mathbb{M}_{\mathcal{P}\text{even}} \mathbb{A}_R^i = \frac{1}{2} ([\mathbb{M}_{\mathcal{P}\text{even}}, \mathbb{A}_R^i] + \{\mathbb{M}_{\mathcal{P}\text{even}}, \mathbb{A}_R^i\}), \\ \mathbb{A}_R^i \cdot \mathbb{M}_{\mathcal{P}\text{odd}} &\stackrel{\text{def}}{=} -\mathbb{A}_R^i \mathbb{M}_{\mathcal{P}\text{odd}} = \frac{1}{2} ([\mathbb{M}_{\mathcal{P}\text{odd}}, \mathbb{A}_R^i] - \{\mathbb{M}_{\mathcal{P}\text{odd}}, \mathbb{A}_R^i\}), \end{aligned} \quad (1)$$

which is akin to left- and right- multiplying $N \times N$ matrices.

From eqs. (1), we see that the diagonal $U(N)$ group acts by commutation with the \mathbb{M} matrices, whatever their behaviour by \mathcal{P} ; the \mathbb{M} 's lie in the adjoint representation of this diagonal $U(N)$.

The expressions in terms of commutators ($[,]$) and anticommutators ($\{ , \}$) have been kept in eq. (1) for the reader to make an easy link with scalars as bound states of fermions (quarks), as evoked in the conclusion.

At the level of the group, let $\mathcal{U}_L \times \mathcal{U}_R$ be a finite transformation of the chiral group; we have

$$\begin{aligned}\mathcal{U}_L \times \mathcal{U}_R \cdot \mathbb{M}_{\mathcal{P}even} &= \mathcal{U}_L^{-1} \mathbb{M}_{\mathcal{P}even} \mathcal{U}_R, \\ \mathcal{U}_L \times \mathcal{U}_R \cdot \mathbb{M}_{\mathcal{P}odd} &= \mathcal{U}_R^{-1} \mathbb{M}_{\mathcal{P}odd} \mathcal{U}_L.\end{aligned}\quad (2)$$

Note that this action only coincides with the usual action of the chiral group in the σ -model for the \mathcal{P} -even scalars, while “left” and “right” are swapped in the action on the \mathcal{P} -odd scalars.

3 The group $SU(2)_L \times U(1)$.

The generators of the “generic” $SU(2)$ we take as

$$\mathbb{T}^3 = \frac{1}{2} \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}, \quad \mathbb{T}^+ = \begin{pmatrix} 0 & \mathbb{I} \\ 0 & 0 \end{pmatrix}, \quad \mathbb{T}^- = \begin{pmatrix} 0 & 0 \\ \mathbb{I} & 0 \end{pmatrix}. \quad (3)$$

The \mathbb{I} 's in eq. (3) stand for $N/2 \times N/2$ unit matrices (we require $\mathbb{T}^- = (\mathbb{T}^+)^\dagger$, such that the unit matrices are chosen to have the same dimension). \mathbb{T}^+ and \mathbb{T}^- are respectively $(\mathbb{T}^1 + i \mathbb{T}^2)$ and $(\mathbb{T}^1 - i \mathbb{T}^2)$. A “left” generic $SU(2)$ is defined accordingly.

The $U(1)$ of hypercharge, non-diagonal, but which commutes with $SU(2)_L$ is defined by its generator $(\mathbb{Y}_L, \mathbb{Y}_R)$, with

$$\begin{aligned}\mathbb{Y}_L &= \frac{1}{6} \mathbb{I}, \\ \mathbb{Y}_R &= \mathbb{Q}_R,\end{aligned}\quad (4)$$

where $\mathbb{Q} = (\mathbb{Q}_L, \mathbb{Q}_R)$ is the charge operator and \mathbb{I} is the unit $N \times N$ matrix.

The Gell-Mann-Nishijima relation

$$\mathbb{Y} = \mathbb{Q} - \mathbb{T}_L^3, \quad (5)$$

to be understood as

$$(\mathbb{Y}_L, \mathbb{Y}_R) = (\mathbb{Q}_L, \mathbb{Q}_R) - (\mathbb{T}_L^3, 0), \quad (6)$$

is verified, for its left and right projections, by the definitions (3,4) above when the charge operator \mathbb{Q} is diagonal with

$$\mathbb{Q}_L = \mathbb{Q}_R = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}. \quad (7)$$

The “alignment” of the electroweak subgroup inside the chiral group is controlled by a unitary matrix, (\mathbb{R}, \mathbb{R}) , acting diagonally, with

$$\mathbb{R} = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & \mathbb{K} \end{pmatrix}, \quad (8)$$

where \mathbb{K} is a $N/2 \times N/2$ unitary matrix of rotation [3, 4]. The ‘rotated’ electroweak group is then the one with generators

$$\mathbb{R}^\dagger \mathbb{T} \mathbb{R}; \quad (9)$$

in practice, this rotation only acts on the \mathbb{T}^\pm generators.

4 Quadruplet scalar representations of $SU(2)_L \times U(1)$.

Because electroweak interactions also violate parity, the representations of the corresponding group of symmetry mix states of different parities, ‘scalars’ and ‘pseudoscalars’. The representations are of two types, \mathcal{P} -even and \mathcal{P} -odd, according to their transformation properties by the parity changing operator \mathcal{P} already mentioned in section 2.

In the same way (see eq. (1)) as we wrote the action of the chiral group on scalar fields represented by $N \times N$ matrices \mathbb{M} , we define the action of its $SU(2)_L$ subgroup, to which we add the action of the electric charge \mathbb{Q} according to:

$$\mathbb{Q} \cdot \mathbb{M} = [\mathbb{M}, \mathbb{Q}]; \quad (10)$$

it acts by commutation because it is a diagonal operator (see section 2).

We shall now build a very special type of representations of the ‘generic’ $SU(2)_L \times U(1)$ group defined in eqs. (3,4). We write them in the form $(\mathbb{M}^0, \tilde{\mathbb{M}})$, where the \mathbb{M} ’s are still $N \times N$ matrices; $\tilde{\mathbb{M}}$ stand for the sets $\{\mathbb{M}^1, \mathbb{M}^2, \mathbb{M}^3\}$ or $\{\mathbb{M}^3, \mathbb{M}^+, \mathbb{M}^-\}$ with $\mathbb{M}^+ = (\mathbb{M}^1 + i\mathbb{M}^2)/\sqrt{2}$, $\mathbb{M}^- = (\mathbb{M}^1 - i\mathbb{M}^2)/\sqrt{2}$.

Let us consider quadruplets of the form

$$(\mathbb{M}^0, \mathbb{M}^3, \mathbb{M}^+, \mathbb{M}^-) = \left[\frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{D} & 0 \\ 0 & \mathbb{D} \end{pmatrix}, \frac{i}{\sqrt{2}} \begin{pmatrix} \mathbb{D} & 0 \\ 0 & -\mathbb{D} \end{pmatrix}, i \begin{pmatrix} 0 & \mathbb{D} \\ 0 & 0 \end{pmatrix}, i \begin{pmatrix} 0 & 0 \\ \mathbb{D} & 0 \end{pmatrix} \right], \quad (11)$$

where \mathbb{D} is a real $N/2 \times N/2$ matrix.

The action of $SU(2)_L \times U(1)$ on these quadruplets is defined by its action on each of the four components, as written in eqs. (1,10). It turns out that it can be rewritten in the form (the Latin indices i, j, k run from 1 to 3):

$$\begin{aligned} \mathbb{T}_L^i \cdot \mathbb{M}_{\mathcal{P}\text{even}}^j &= -\frac{i}{2} \left(\epsilon_{ijk} \mathbb{M}_{\mathcal{P}\text{even}}^k + \delta_{ij} \mathbb{M}_{\mathcal{P}\text{even}}^0 \right), \\ \mathbb{T}_L^i \cdot \mathbb{M}_{\mathcal{P}\text{even}}^0 &= \frac{i}{2} \mathbb{M}_{\mathcal{P}\text{even}}^i; \end{aligned} \quad (12)$$

and

$$\begin{aligned} \mathbb{T}_L^i \cdot \mathbb{M}_{\mathcal{P}\text{odd}}^j &= -\frac{i}{2} \left(\epsilon_{ijk} \mathbb{M}_{\mathcal{P}\text{odd}}^k - \delta_{ij} \mathbb{M}_{\mathcal{P}\text{odd}}^0 \right), \\ \mathbb{T}_L^i \cdot \mathbb{M}_{\mathcal{P}\text{odd}}^0 &= -\frac{i}{2} \mathbb{M}_{\mathcal{P}\text{odd}}^i. \end{aligned} \quad (13)$$

The charge operator acts indifferently on \mathcal{P} -even and \mathcal{P} -odd matrices by:

$$\begin{aligned} \mathbb{Q} \cdot \mathbb{M}^i &= -i \epsilon_{ij3} \mathbb{M}^j, \\ \mathbb{Q} \cdot \mathbb{M}^0 &= 0, \end{aligned} \quad (14)$$

and the action of the $U(1)$ generator \mathbb{Y} follows from eq. (5).

We see that we deal now with 4-dimensional representations of $SU(2)_L \times U(1)$. In the basis of any such representation, the generators of the electroweak group can be rewritten as 4×4 matrices. (This is also the case for the generators of the diagonal $SU(2)$ evoked above, which will be exploited in the next section concerning the quantization of the electric charge).

We shall restrict below to this type of representations (11).

They decompose into “symmetric” representations, corresponding to $\mathbb{D} = \mathbb{D}^\dagger$, and “antisymmetric” ones for which $\mathbb{D} = -\mathbb{D}^\dagger$.

There are $N/2(N/2+1)/2$ independent real symmetric \mathbb{D} matrices; hence, the sets of “even” and “odd” symmetric quadruplet representations of the type (11) both have dimension $N/2(N/2+1)/2$. Similarly, the antisymmetric ones form two sets of dimension $N/2(N/2-1)/2$.

If $(\mathbb{M}^0, \vec{\mathbb{M}})$ is a representation of the “generic” $SU(2)_L \times U(1)$ of eqs. (3,4), $(\mathbb{R}^\dagger \mathbb{M}^0 \mathbb{R}, \mathbb{R}^\dagger \vec{\mathbb{M}} \mathbb{R})$ is a representation of the “rotated” group of eq. (9); it is called hereafter a “rotated” representation.

Every representation above is a reducible representation of $SU(2)_L$ and is the sum of two (complex) representations of spin 1/2. This makes it isomorphic to the standard scalar set of the Glashow-Salam-Weinberg model [1].

Now, if we consider the transformation properties by the diagonal $SU(2)$, all $\vec{\mathbb{M}}$ ’s are (spin 1) triplets, lying in the adjoint representation, while all \mathbb{M}^0 ’s are singlets.

To ease the link with physics, let us make one more step in the reshuffling of our quadruplets. By summing or subtracting the 2 representations, \mathcal{P} -even and \mathcal{P} -odd, corresponding to the same set of four \mathbb{M} matrices, one can form 2 other representations; the action of the group rewrites, using eqs. (12,13):

$$\begin{aligned} \mathbb{T}_L^i \cdot (\mathbb{M}_{\mathcal{P}\text{even}}^j + \mathbb{M}_{\mathcal{P}\text{odd}}^j) &= -\frac{i}{2} \left(\epsilon_{ijk} (\mathbb{M}_{\mathcal{P}\text{even}}^k + \mathbb{M}_{\mathcal{P}\text{odd}}^k) + \delta_{ij} (\mathbb{M}_{\mathcal{P}\text{even}}^0 - \mathbb{M}_{\mathcal{P}\text{odd}}^0) \right), \\ \mathbb{T}_L^i \cdot (\mathbb{M}_{\mathcal{P}\text{even}}^0 + \mathbb{M}_{\mathcal{P}\text{odd}}^0) &= \frac{i}{2} (\mathbb{M}_{\mathcal{P}\text{even}}^i - \mathbb{M}_{\mathcal{P}\text{odd}}^i); \end{aligned} \quad (15)$$

$$\begin{aligned} \mathbb{T}_L^i \cdot (\mathbb{M}_{\mathcal{P}\text{even}}^j - \mathbb{M}_{\mathcal{P}\text{odd}}^j) &= -\frac{i}{2} \left(\epsilon_{ijk} (\mathbb{M}_{\mathcal{P}\text{even}}^k - \mathbb{M}_{\mathcal{P}\text{odd}}^k) + \delta_{ij} (\mathbb{M}_{\mathcal{P}\text{even}}^0 + \mathbb{M}_{\mathcal{P}\text{odd}}^0) \right), \\ \mathbb{T}_L^i \cdot (\mathbb{M}_{\mathcal{P}\text{even}}^0 - \mathbb{M}_{\mathcal{P}\text{odd}}^0) &= \frac{i}{2} (\mathbb{M}_{\mathcal{P}\text{even}}^i + \mathbb{M}_{\mathcal{P}\text{odd}}^i). \end{aligned} \quad (16)$$

It is convenient to rewrite

$$(\mathbb{M}_{\mathcal{P}\text{even}} + \mathbb{M}_{\mathcal{P}\text{odd}}) = \mathbb{S}, \quad (17)$$

and

$$(\mathbb{M}_{\mathcal{P}\text{even}} - \mathbb{M}_{\mathcal{P}\text{odd}}) = \mathbb{P}, \quad (18)$$

eq. (17) corresponding to a scalar state \mathbb{S} , and eq. (18) to a pseudoscalar state \mathbb{P} . Thus, of those two new representations, the first is of the type

$$(\mathbb{M}^0, \vec{\mathbb{M}}) = (\mathbb{S}^0, \vec{\mathbb{P}}) \quad (19)$$

and the second of the type

$$(\mathbb{M}^0, \vec{\mathbb{M}}) = (\mathbb{P}^0, \vec{\mathbb{S}}); \quad (20)$$

both have scalar and pseudoscalar entries, each entry having a definite *P quantum number* (we attribute to scalars the parity $P = +1$ and to pseudoscalars the parity $P = -1$). Among the “symmetric” $(\mathbb{S}^0, \vec{\mathbb{P}})$ representations lies the one corresponding to $\mathbb{D} = \mathbb{I}$ and which thus includes the scalar $U(N)$ singlet: it is hereafter identified with the Higgs boson H and the corresponding representation with the usual scalar 4-plet of the Standard Model. From now onwards, we shall work with the representations (19) and (20).

By hermitian conjugation a “symmetric” $(\mathbb{M}^0, \vec{\mathbb{M}})$ representation gives $(\mathbb{M}^0, -\vec{\mathbb{M}})$; an “antisymmetric” representation gives $(-\mathbb{M}^0, \vec{\mathbb{M}})$; the representations (19) and (20) are consequently representations of given CP (charge conjugation \times parity): “symmetric” $(\mathbb{S}^0, \vec{\mathbb{P}})$ ’s and “antisymmetric” $(\mathbb{P}^0, \vec{\mathbb{S}})$ ’s are CP -even, while “symmetric” $(\mathbb{P}^0, \vec{\mathbb{S}})$ ’s and “antisymmetric” $(\mathbb{S}^0, \vec{\mathbb{P}})$ ’s are CP -odd.

5 The $SU(2) \times U(1)$ invariant Lagrangian for scalar fields.

To every representation (11) is associated a unique quadratic form invariant by any $SU(2)_L \times U(1)$ transformation:

$$\mathcal{I} = \mathbb{M}^0 * (\mathbb{M}^0)^\dagger + \vec{\mathbb{M}} * (\vec{\mathbb{M}})^\dagger; \quad (21)$$

the “*” product is *not* meant in the sense of the usual multiplication of matrices but in the sense of the product of fields as functions of space-time. $\vec{\mathbb{M}} * (\vec{\mathbb{M}})^\dagger$ stands for $\sum_{i=1,2,3} \mathbb{M}^i * (\mathbb{M}^i)^\dagger$.

Once we have the action of the (gauge) group and a quadratic invariant, we can immediately write a gauge invariant electroweak Lagrangian for the $2N^2$ scalar and pseudoscalar fields. It includes *a priori* $N^2/2$ independent mass scales.

$$\mathcal{L} = \sum_{\text{all reps } \mathcal{R}} \frac{1}{2} \left(D_\mu \mathbb{M}_\mathcal{R}^0 * (D^\mu \mathbb{M}_\mathcal{R}^0)^\dagger + D_\mu \vec{\mathbb{M}}_\mathcal{R} * (D^\mu \vec{\mathbb{M}}_\mathcal{R})^\dagger - m_\mathcal{R}^2 (\mathbb{M}_\mathcal{R}^0 * (\mathbb{M}_\mathcal{R}^0)^\dagger + \vec{\mathbb{M}}_\mathcal{R} * (\vec{\mathbb{M}}_\mathcal{R})^\dagger) \right). \quad (22)$$

D_μ in eq. (22) is the covariant derivative with respect to $SU(2)_L \times U(1)$.

6 Quantization of the electric charge.

We have already mentioned the importance of the “diagonal” $SU(2)$ group. The 4-dimensional representations (11) of $SU(2)_L \times U(1)$ being also representations of this group of symmetry, its generators $\mathbb{T}^3, \mathbb{T}^\pm$, when acting in the 4-dimensional vector space of which (11) form a basis, can be represented as 4×4 matrices $\tilde{\mathbb{T}}^3, \tilde{\mathbb{T}}^\pm$; explicitly:

$$\tilde{\mathbb{T}}^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\sqrt{2} & 0 & 0 \end{pmatrix}, \quad \tilde{\mathbb{T}}^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{2} \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{\mathbb{T}}^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (23)$$

That the first line in any of the three above matrices identically vanishes is the translation of the already mentioned fact that the first entry (\mathbb{M}^0) of the representations (11) are

singlets by the diagonal $SU(2)$, while the three other entries (\vec{M}) form a triplet in the adjoint representation.

In the same way, the charge operator becomes \tilde{Q} :

$$\tilde{Q} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

So, it occurs that \tilde{Q} is identical with \tilde{T}^3 and that we have the commutation relation

$$[\tilde{T}^+, \tilde{T}^-] = 2 \tilde{T}^3 = 2 \tilde{Q}. \quad (25)$$

\tilde{Q} being an $SU(2)$ generator, its eigenvalues, hence the electric charges of the representations (11), are quantized.

7 Summary of the group structure.

It is instructive to summarize the group structure that we have been dealing with. Everything occurs “inside” the chiral $U(N)_L \times U(N)_R$ group. The “generic” $SU(2)_L$ group is “aligned” with the chiral group and lies of course in the “left” part of it. The “rotated” $SU(2)_L$ group is deduced from the latter by a unitary matrix of rotation (Cabibbo, Kobayashi-Maskawa). The $U(1)$ group is not diagonal, and thus extends in a non-symmetrical way on both the “left” and “right” sides of the chiral group. The diagonal $SU(2)$, built from the “rotated” $SU(2)_L$ and its ‘right’ image, includes the electric charge \tilde{Q} as one of its generators. The latter is also, according to the Gell-Mann-Nishijima equation (6), the diagonal part of the above mentioned $U(1)$ group.

8 Particles: a few brief remarks.

The “standard” scalar 4-plet of the Glashow-Salam-Weinberg model is identified with the symmetric $(\mathbb{S}^0, \vec{\mathbb{P}})$ representation including the scalar $U(N)$ singlet, represented by the $N \times N$ unit matrix. The latter can be chosen as the only diagonal $N \times N$ matrix with a non vanishing trace, and be unambiguously defined as the Higgs boson. It is the only field supposed to have a non-vanishing vacuum expectation value.

Any linear combination of the above representations also being a representation, only physical observation can guide us towards the determination of what are the observed electroweak eigenstates. Mixing matrices link physical states with the (rotated) representations displayed above; they can *a priori* depend on new parameters, differing or not from the angles and phases characterizing, in the rotation matrix \mathbb{R} , the alignment of the electroweak group inside the chiral group. Combining representations of both types $(\mathbb{S}^0, \vec{\mathbb{P}})$ and $(\mathbb{P}^0, \vec{\mathbb{S}})$ seems however not desired since it would mix states of different parities.

Any state produced by strong interactions is a combination of electroweak eigenstates, which evolve and decay according to the dynamics of electroweak interactions if no strong channel is allowed for the decay of the initial state.

Only those representations of the $(\mathbb{S}^0, \vec{\mathbb{P}})$ type, and for which the scalar entry has a non-vanishing component on the Higgs boson, will witness leptonic decays of their three $\vec{\mathbb{P}}$ entries; indeed, only for those representations will the kinetic term in the Lagrangian (22) include a $\langle H \rangle \sigma^\mu \partial_\mu \mathbb{P}$ coupling, where σ^μ is a gauge field; the direct coupling of the latter to leptons will trigger the leptonic decay of the pseudoscalar \mathbb{P} . In the same way, we deduce that, under our hypotheses, scalar mesons never leptonically decay: by the action of the group, a scalar is connected either to a pseudoscalar, which is supposed to have a vanishing vacuum expectation value, or to another scalar; but the latter is always one with a vanishing vacuum expectation value since the Higgs boson can only be reached by acting with the group on a pseudoscalar.

Semi-leptonic decays between states of the same parity can only occur between the members of the (diagonal) $SU(2)$ triplet of a given quadruplet, since the gauge group only connects the entries of a given representation; indeed, the kinetic term includes couplings of the type $\mathbb{P}_1 \sigma^\mu \partial_\mu \mathbb{P}_2$, with the gauge field σ_μ giving leptons as before. In particular, a (diagonal) $SU(2)$ singlet like a \mathbb{P}^0 or a \mathbb{S}^0 never semi-leptonically decays into another meson of the same parity.

So, the customary attribution of CP quantum numbers and the presence or not of semi-leptonic decays makes that the “short-lived” neutral kaon, which is not observed to decay semi-leptonically, is most probably the $SU(2)$ “singlet” of an “antisymmetric” $(\mathbb{P}^0, \vec{\mathbb{S}})$ representation ($CP = +1$), while the neutral pion, if thought of as aligned with the corresponding “strong” eigenstate, and the “long-lived” kaon should *a priori* be looked for in “antisymmetric” $(\mathbb{S}^0, \vec{\mathbb{P}})$ representations ($CP = -1$).

The simple remarks above already have for consequence that, would N be equal to 4, the neutral electroweak pion and the long-lived neutral kaon cannot be both pure CP eigenstates: there is indeed, in this case, only one “antisymmetric” $(\mathbb{S}^0, \vec{\mathbb{P}})$ representation. In a world with $N = 4$, CP has to be violated at the level of the eigenstates themselves. If $N = 6$, there are three such representations and consequently three different $\vec{\mathbb{P}}$ triplets can include a pure CP -odd neutral electroweak eigenstate. But it is not enough to settle the case in favour of $N \geq 6$, because, as already stated, leptonic decays are forbidden for these “antisymmetric” representations, in which no entry has been allowed to have a non-vanishing vacuum expectation value (an *electroweakly* created “antisymmetric” pseudoscalar cannot decay into leptons). This is in particular valid for the charged partners of the neutral fields under concern, which are automatically given the same electroweak mass scale as the latter because of the uniqueness of the quadratic invariant (only electromagnetic effects are here expected to lift the mass degeneracy between them). Hence, we are facing an alternative:

- either the vacuum structure of the theory is more complicated than has been supposed, in particular “antisymmetric” scalars are allowed to also have a non-vanishing vacuum expectation value;
- or the experimental evidence of leptonic decays for, say, the charged electroweak kaons, is the hint that the physically observed electroweak eigenstates include an admixture of the CP -even “symmetric” representation which includes the Higgs boson, *i.e.* they are *not* pure CP eigenstates. This suggests that, whatever N , CP violation already occurs at the level of the eigenstates, and not only at that of the interactions themselves for $N \geq 6$ [4].

9 Conclusion. Outlook.

We have so far built a gauge theory of mesons without any reference to their eventual composite nature, and have nowhere mentioned the existence of fermions other than leptons. However, the reader can easily check all the computations and results by sandwiching the matrices \mathbb{M} between a N -vector Ψ of “quarks” in the fundamental representation of $U(N)$, an its conjugate $\bar{\Psi}$, and by introducing a γ_5 in the definition of all \mathbb{P} pseudoscalar states. The “left” and “right” generators are respectively given a $(1 - \gamma_5)/2$ or a $(1 + \gamma_5)/2$ projectors when acting on the fermions, and the laws of transformations of the latter determine those of the mesons; *all the group actions on $J = 0$ fields that we have written can be uniquely and straightforwardly deduced from the action on fermions when the former are written as scalar or pseudoscalar diquark operators.* Then, the occurrence of commutators and anticommutators in eqs. (1,10) becomes natural.

In particular, by taking \mathbb{K} in the rotation matrix of eq. (8) equal to the Cabibbo matrix [3], the reader will recover the eight electroweak representations exhibited in ref. [5]. The Cabibbo angle is also taken there to control the linear combinations of the (rotated) representations which form the physical electroweak eigenstates (the alignment between “strong” and “electroweak” charged eigenstates supposed in ref. [5] makes this the more likely).

The quarks could then be thought of as convenient technical entities only, which could be forgotten. We have however not considered here quantum effects, like in particular the decays of the neutral pion into two photons. It is shown in refs. [6, 7, 5] that taking into account the compositeness of the mesons in building a quantum theory correctly reproduces those, too. The three works just quoted start from the fermionic point of view and perform a determination of the observed electroweak eigenstates in terms of the above representations; they go also further in the study of their physical interactions and decays.

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